コヒーレント状態による 単位作用素の分解の強収束性 Quantum Foundations 2024 3/11

なみきりょう

$$
|\phi\rangle = \frac{1}{\pi} \int_{\alpha \in \mathbb{C}} |\alpha\rangle \langle \alpha | \phi \rangle d^2 \alpha \qquad \text{c.f.} \quad |\phi\rangle = \sum_{n=0}^{\infty} |n\rangle \langle n | \phi \rangle
$$

$$
|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle, \quad \alpha \in \mathbb{C} \qquad I \stackrel{\text{s}}{=} \sum_{n=0}^{\infty} |n\rangle \langle n|
$$
arXiv:2402 08317

コヒーレント状態

最小不確定状態、レーザー光

•消滅演算子の固有状態

$$
\cdot \, |\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \, |n\rangle, \quad \alpha \in \mathbb{C} \, .
$$

$$
a | \alpha \rangle = \alpha | \alpha \rangle \qquad [a, a^{\dagger}] = 1
$$

$$
[a, a^{\dagger}] = \mathbf{1}
$$

•積分表示の単位の分解

$$
I = \int_{\alpha \in \mathbb{C}} \frac{1}{\pi} |\alpha\rangle \langle \alpha| d^2 \alpha
$$

I s = ∞ ∑ *n*=0 c.f. 完全性条件 |*n*⟩⟨*n*| ランダムユニタリ

積分表示の単位作用素の分解

$$
I = \int_{\alpha \in \mathbb{C}} \frac{1}{\pi} |\alpha\rangle \langle \alpha| d^2\alpha
$$

 $I \stackrel{s}{=}$ ∞ ∑ $n=0$ c.f. 完全性条件 |*n*⟩⟨*n*| ランダムユニタリ

たいてい弱収束、弱い意味で、との注意書きがある、なぜか?

$$
\int \mathbb{S} \mathbb{S} \cup \hat{\mathbb{S}} \mathbb{R} : \qquad \langle \psi | \phi \rangle = \frac{1}{\pi} \int_{\alpha \in \mathbb{C}} \langle \psi | \alpha \rangle \langle \alpha | \phi \rangle \mathrm{d}^2 \alpha
$$

- 利点:ベクトル値あるいは作用素値の積分を考えずに済む
- 欠点:通常の分解(展開)が使えない

$$
|\phi\rangle = \frac{1}{\pi} \int_{\alpha \in \mathbb{C}} |\alpha\rangle \langle \alpha | \phi \rangle d^2 \alpha \qquad \text{c.f.} \quad |\phi\rangle = \sum_{n=0}^{\infty} |n\rangle \langle n | \phi \rangle
$$

形式的な計算 $\overline{ }$ ∞ " 式的な計算 |**d2**/C| **l**

. . . .

n!*m*! Klauder, Ann. Phys. 11, 123 (1960) **2** *Mandel and Wolf,*
*Continel coherence a*¹
btical coherence and Quantum optic *a*
*i C*¹¹ Mandel and Wolt,
Optical coherence and Quantum optics (1995)

$$
I = \int_{\alpha \in \mathbb{C}} \frac{1}{\pi} |\alpha\rangle \langle \alpha| d^2 \alpha \qquad |\alpha\rangle = e^{-|\alpha|}
$$

n=0

m=0

$$
|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle, \quad \alpha \in \mathbb{C}
$$

積分と無限和を交換していいとする

$$
\int |\alpha\rangle\langle\alpha| d^2\alpha = \int_{\substack{n=0 \ n\neq 0}}^{\infty} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{\alpha^n \bar{\alpha}^m}{\sqrt{n!m!}} |n\rangle\langle m| d^2\alpha
$$

\n
$$
= \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \int_{\substack{n=0 \ n\neq 0}}^{\infty} \left(e^{-|\alpha|^2} \frac{\alpha^n \bar{\alpha}^m}{\sqrt{n!m!}} \right) d^2\alpha |n\rangle\langle m| = \pi \sum_{n=0}^{\infty} |n\rangle\langle n|
$$

\n
$$
\lim_{n\to\infty} \sum_{m=0}^{\infty} \sum_{m=0}^{\infty} \frac{\alpha^n \bar{\alpha}^m}{\sqrt{n!m!}} d^2\alpha |n\rangle\langle m| = \pi \sum_{n=0}^{\infty} |n\rangle\langle n|
$$

n=0

m=0

形式的な計算 $\overline{ }$ ∞ " 式的な計算 |**d2**/C| **l**

,,,,,,

n!*m*! Klauder, Ann. Phys. 11, 123 (1960) **2** *Mandel and Wolf,*
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btical coherence and Quantum optic *a*
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Optical coherence and Quantum optics (1995)

$$
I = \int_{\alpha \in \mathbb{C}} \frac{1}{\pi} |\alpha\rangle \langle \alpha| d^2 \alpha \qquad |\alpha\rangle = e^{-|\alpha|^2/2}
$$

n=0

m=0

"α*|*ϕ!*d*²^α $e^{-|\alpha|/2}$ $\sum \frac{1}{\sqrt{n!}} |n\rangle, \quad \alpha \in \mathbb{C}$ ²*/*² α*ⁿ* $\langle \alpha \rangle = e^{-|\alpha|^2/2}$ $\frac{\alpha}{\alpha}$ $\alpha \in \mathbb{C}$
 $\sqrt{n!}$ $\left\{n\right\}, \alpha \in \mathbb{C}$ ∞ ∑ *n*=0 *αn n*! $|n\rangle, \quad \alpha \in \mathbb{C}$

積分と無限和を交換していいとする

$$
\int |\alpha\rangle\langle\alpha| d^2\alpha = \int_{n=0}^{1} e^{-|\alpha|^2} \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \frac{\alpha^n \bar{\alpha}^m}{\sqrt{n!m!}} |n\rangle\langle m| d^2\alpha
$$
\n
$$
= \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \int_{\text{min}}^{\infty} \left(e^{-|\alpha|^2} \frac{\alpha^n \bar{\alpha}^m}{\sqrt{n!m!}} \right) d^2\alpha |n\rangle\langle m| = \pi \sum_{n=0}^{\infty} |n\rangle\langle n|
$$
\n
$$
\text{max}(\bar{\alpha}^2) = \pi \sum_{n=0}^{\infty} |\alpha| \sum_{n=0}^{\infty} \sum_{m=0}^{\infty} \int_{\text{min}}^{\infty} \left(e^{-|\alpha|^2} \frac{\alpha^n \bar{\alpha}^m}{\sqrt{n!m!}} \right) d^2\alpha |n\rangle\langle m| = \pi \sum_{n=0}^{\infty} |n\rangle\langle n|
$$
\n
$$
\text{max}(\bar{\alpha}^2) = \pi \delta_{n,m}
$$

n=0

m=0

形式的な計算 $\overline{ }$ ∞ " 式的な計算 |**d2**/C| **l**

n!*m*! Klauder, Ann. Phys. 11, 123 (1960) Mande
ممثلا *a*¹
btical coherence and Quantum optic " *Anadder, Anni. Γιιγέ*
Mandel and Wolf, *a*
*i C*¹¹ Mandel and Wolt,
Optical coherence and Quantum optics (1995)

$$
I = \int_{\alpha \in \mathbb{C}} \frac{1}{\pi} | \alpha \rangle \langle \alpha | d^2 \alpha
$$

$$
| \alpha \rangle = e^{-| \alpha |^2 / 2}
$$

n=0

m=0

"α*|*ϕ!*d*²^α $e^{-|\alpha|/2}$ $\sum \frac{1}{\sqrt{n!}} |n\rangle, \quad \alpha \in \mathbb{C}$ ²*/*² α*ⁿ* $\langle \alpha \rangle = e^{-|\alpha|^2/2}$ $\frac{\alpha}{\alpha}$ $\alpha \in \mathbb{C}$
 $\sqrt{n!}$ $\left\{n\right\}, \alpha \in \mathbb{C}$ ∞ ∑ *n*=0 *αn n*! $|n\rangle, \quad \alpha \in \mathbb{C}$

積分と無限和を交換していいとする

n=0

m=0

作用素の収束

From the properties (i) and (ii) in Eq. (13), n ≤ k implies

①ノルム収束:通常の収束

 $A = \lim$ $\lim_{n\to\infty} A_n \quad \Leftrightarrow \quad \lim_{n\to\infty} ||A-A_n|| = 0$ \mathcal{L} $|A\phi|$ ≤ 1 $||A|| := \sup$ $||A\phi||$ ∥*ϕ*∥≤1 $||\phi|| := \sqrt{\langle \phi | \phi \rangle}$

$$
I_k = \sum_{n=0}^k |n\rangle \langle n|
$$

' [|]ϕ! − \$% d2α π &' ' ' < & (11) #*I^m* − *Ik*# = " *m* " " " " " ! *n*=*k*+1 *|n*! "*n|* " " " " = 1 コーシー列ではない・・・ 通常のノルムでは収束しない! ⇒

作用素の収束 \blacksquare = \blacksquare and \blacksquare and \blacksquare $\frac{1}{2}$ may concern the following three topologies of operations of operations of $\frac{1}{2}$ \mathbf{E} TEX についてのノート メード

From the properties (i) and (ii) in Eq. (13), n ≤ k implies

①ノルム収束:通常の収束 I. 発表準備

\n
$$
A = \lim_{n \to \infty} A_n \quad \Leftrightarrow \quad \lim_{n \to \infty} \|A - A_n\| = 0
$$
\n

\n\n $\|\mathbf{A}\| := \sup_{\|\phi\| \leq 1} \|A\phi\|$ \n

\n\n $\text{②.} \quad \text{②.} \quad \text{③.} \quad \text{�$

 $A = s$ - lim $= s\text{-} \lim_{n \to \infty} A_n \Leftrightarrow \lim_{n \to \infty} ||A\phi - A_n\phi|| = 0, \ \forall \phi \in \mathcal{H}.$ $\lim_{n\to\infty} A_n \Leftrightarrow \lim_{n\to\infty} ||A\varphi||$ $-A_n$ \mathcal{F} \mathcal{F} \mathcal{F} \mathcal{F} \mathcal{F} $\overline{}$ *l* $\rightarrow \infty$ " \vert " \mathbf{r} 1% $\frac{1}{2}$ $^{\prime}$ " = 1 #*I^m* − *Ik*# = " " " \rightarrow U *m |n*! "*n|* .
-" = 1 #(*I^m* − *Ik*)*|*ϕ! # = $\overline{\mathcal{L}}$ $\left\langle \right\rangle$ $\frac{1}{2}$ $\rightarrow \infty$ [|] | *|* \overline{b} $\mathcal{P}_\mathcal{C}$ $\overline{1}$ $\overline{1}$ $n \, \varphi$ *m* $\overline{}$ $^{\prime}$ $\forall \phi \in \mathcal{H}$ *m*

$$
I_k = \sum_{n=0}^k |n\rangle \langle n| \qquad \qquad |\varphi\rangle = \sum_n a_n |n\rangle \in \mathcal{H} \iff \sum_{n=0}^\infty |a_n|^2 < \infty
$$

 $\Vert \varphi \rangle \Vert = \Big\Vert \sum_{n=1}^{\infty} \vert n \rangle \langle n \vert \vert \varphi \rangle \Big\Vert = \Big\Vert \sum_{n=1}^{\infty} \vert a_n \vert n \rangle \Big\Vert = \sqrt{ \sum_{n=1}^{\infty} \vert a_n \vert^2 } \quad \rightarrow \quad$ $\lvert \lvert n = k + 1 \rvert \qquad \qquad \lvert \lvert n = k + 1 \rvert \qquad \qquad \lvert \lvert n = k + 1 \rvert \qquad \qquad \lvert \lvert n = k + 1 \rvert \qquad \qquad$ $\rightarrow \infty$) $m \to \infty$) R > 0 such that \parallel $=\left\|\sum_{n=k+1}^{\infty} \binom{n}{n} \binom{n}{n} \right\| = \left\|\sum_{n=k+1}^{\infty} \binom{n}{n} \right\|$ コーシー列となる $\begin{array}{c} a_n \mid n \rangle \ +1 \end{array}$ $\sqrt{n} = \kappa + 1$ $+\infty)$ $\|(I_m - I_k)\ket{\varphi}\| =$ $\frac{1}{2}$ $\begin{array}{c} \hline \end{array}$ $\begin{array}{c} \hline \end{array}$ $\begin{array}{c} \hline \end{array}$ $\frac{1}{2}$ \sum *m n*=*k*+1 $|n\rangle \left\langle n\right| |\varphi\rangle$ $\frac{1}{2}$ \parallel \parallel \parallel $\prod_{i=1}^{n}$ = $\frac{1}{2}$ $\begin{array}{c} \hline \end{array}$ $\begin{array}{c} \hline \end{array}$ $\begin{array}{c} \hline \end{array}$ $\frac{1}{2}$ \sum *m n*=*k*+1 $a_n |n\rangle$ $\frac{1}{2}$ \parallel \parallel \parallel $\prod_{i=1}^{n}$ $=\sqrt{\sum_{n=1}^{m}}$ *m n*=*k*+1 $|a_n|^2 \rightarrow 0 \quad (k, m \rightarrow \infty)$ #(*I^m* − *Ik*)*|*ϕ! # = " $\overline{}$ \sim 1 *n*=*k*+1 *|n*! "*n| |*ϕ! \mathcal{L} $\langle \gamma$ " $\overline{\rho}$! λ \parallel $=$ \parallel *aⁿ |n*! $\begin{array}{c} \begin{array}{c} \end{array}$ α \rightarrow 11 α *|an|* ² → 0 (*k,m* → ∞) コーシー $m = 1k$ *J* $|\psi|$.
= " <u>||</u> *m n*=*k*+1 *n* [|]^{*n*}/^{*n*} | $\overline{1}$ \langle " $\frac{1}{2}$ #*I^m* − *Ik*# = " " *n*=*K*+1 *|n*! "*n|* " = 1 #*I* − *Ik*# → 0 (*k* → 0) $\begin{array}{ccc} \n\begin{array}{ccc}\n1 & 1 & 1 & 1 & 1 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
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0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 &$ *[|]*α!"α*[|] ^d*²^α ⁼ *e*−*|*α*[|]* \mathbf{v} √ *n*!*m*! *[|]n*!"*m|d*²^α

 \equiv

|n! "*n|*

シ<u>ー</u>

"

ⁿ→∞ \$% &

= 1 #*I* − *Ik*# → 0 (*k* → 0)

D(*R*)

 $\mathbf{v} = \mathbf{v}$

1 F H C

コーシー列となる、状態に作用させれば収束=強収束!

<mark>状態に作用させれは収束=強収束!</mark>

作用素の収束 \blacksquare = \blacksquare and \blacksquare and \blacksquare $\frac{1}{2}$ may concern the following three topologies of operations of operations of $\frac{1}{2}$ \mathbf{E} n→∞ ⁿ→∞ "A^φ [−] ^Anφ" = 0, [∀]^φ [∈] ^H.

The property (iv) follows from (i) and (ii).

what lengthy process (see Appendix C), we have

From the properties (i) and (ii) in Eq. (13), n ≤ k implies

 \blacksquare

where we used the property (iv) in Eq. (13) to obtain the

①ノルム収束:通常の収束

$$
A = \lim_{n \to \infty} A_n \quad \Leftrightarrow \quad \lim_{n \to \infty} \|A - A_n\| = 0
$$

$$
\|A\| := \sup_{\|\phi\| \le 1} \|A\phi\|
$$

②強収束:作用させた状態が収束

$$
A = s \cdot \lim_{n \to \infty} A_n \Leftrightarrow \lim_{n \to \infty} ||A\phi - A_n\phi|| = 0, \ \forall \phi \in \mathcal{H}.
$$

:行列要素が収束 We say (An) converges to A in the uniform operator \mathcal{A} in the uniform operator operator operator \mathcal{A} $\sum_{i=1}^n \sum_{i=1}^n \sum_{j=1}^n \sum_{j$ ③弱収束:行列要素が収束

$$
A = w - \lim_{n \to \infty} A_n \Leftrightarrow \lim_{n \to \infty} \langle \psi | A - A_n | \phi \rangle = 0, \ \forall \phi, \psi \in \mathcal{H}.
$$

We say (An) converges to A in the strong operator topology (or the topology of H) if A = s- lim Aⁿ ⇔ lim ⁿ→∞ "A^φ [−] ^Anφ" = 0, [∀]^φ [∈] ^H. =# ⁿ→∞ "^A [−] ^An" = 0, (9) where the operator norm is defined by sup\$φ\$≤¹ "Aφ". Our primary goal is to show the strong convergence: ⁿ→∞ \$% d2α & last inequality. Let be & > 0. Since ϕ ∈ H we can select a sufficiently large ^K [∈] ^N such that it holds for ^k [≥] ^K ' ' [|]ϕ! − \$% |α|≤r |α! #α|ϕ! ' ' < & (11) #) ¹ [−] ^In(r2) ≤) ¹ [−] ^Ik(r2) ① ⇒ ② ⇒ ③ 通常 強 弱 *I s* ⁼ [∫]*α*∈ℂ 1 *π* |*α*⟩⟨*α*|d2 *α* ? Schwartz ⟨*^ψ* [|]*^A* [−] *An* [|]*ϕ*⟩ [≤] [∥]*ψ*∥∥(*^A* [−] *An*)*ϕ*[∥] 弱収束でいいのでしょうか?

主要結果:強収束します

arXiv:2402.0831

$$
I = s\text{-} \lim_{r \to \infty} \int_{|\alpha| \le r} \frac{|\alpha\rangle\,\langle \alpha|}{\pi} d^2\alpha
$$

D(r)

$$
|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle,
$$

$$
\int_{D(R)} |\alpha\rangle\langle\alpha|\,\varphi\rangle d^2\alpha = \int_{D(R)} e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle\langle\alpha|\varphi\rangle d^2\alpha
$$
\n
$$
= \sum_{n=0}^{\infty} \left(\int_{D(R)\atop\partial\hbar} e^{-|\alpha|^2/2} \frac{\alpha^n}{\sqrt{n!}} \langle\alpha|\varphi\rangle d^2\alpha \right) |n\rangle
$$
\n
$$
\sqrt{\sum_{n=0}^{\infty} \int_{\hbar} \langle\alpha|\varphi\rangle d^2\alpha}
$$
\n
$$
\int \begin{pmatrix} f \\ g \\ h \\ \vdots \end{pmatrix} = \begin{pmatrix} \int f \\ g \\ f \\ \vdots \end{pmatrix}
$$
\n
$$
\sum_{n=0}^{\infty} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} \langle\alpha|\varphi\rangle d^2\alpha
$$
\n
$$
\int_{\hbar} \langle\alpha|\varphi\rangle d^2\alpha
$$
\n
$$
\int_{\hbar} \langle\alpha|\varphi\rangle d^2\alpha = \int_{\hbar} \langle\alpha|\varphi\rangle d^2\alpha
$$

\n $\begin{aligned}\n \mathbf{E} \mathbf{H} \quad D(r) &:= \{ \alpha \in \mathbb{C} \mid \alpha \le r \} \\ \text{(i)} \quad \sum_{n=0}^{\infty} a_n ^2 \alpha ^{2n} < \infty, \quad \alpha \in D(r), \\ \text{(ii)} \quad \int_{D(r)} \sum_{n=0}^{\infty} a_n ^2 \alpha ^{2n} d^2 \alpha < \infty.\n \end{aligned}$ \n	\n $\sum_{n=0}^{\infty} \int_{D(r)} \left(\sum_{n=0}^{\infty} a_n \alpha^n n \rangle \right) d^2 \alpha = \sum_{n=0}^{\infty} \left(\int_{D(r)} a_n \alpha^n d^2 \alpha \right) n \rangle.$ \n
---	--

"

<u>(ii) #1</u>

"

n=*k*+1

"

integration are commutable due to the linearity of inte-

n=*k*+1

(F9)

Now, is considered the following integration integration. The following integration: $\mathcal{L}_{\mathcal{A}}$

 U sing Schwartz's inequality, we can show the power series in equality, we can show the power series \mathcal{U}

"

n=*k*+1

積分と和の交換 $\tau \pm \tau$ n見刀 ⊂ 1´H V ズ] ズ mal basis on H. Let be D(r) := { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C =
{ α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C =
} $\mathbf{H} \mathbf{H} \mathbf{I}$ is $\mathbf{I} \mathbf{I}$ necessitate in the Bochner integration and the Bochner integration integration and the monotone convergence theorem. Therefore, we can exchange the order of the integration \mathbb{R}^n . The integration of the integration of the integration $\mathbf{I} \equiv \boldsymbol{\Delta} \boldsymbol{\lambda}$ is the summation $\boldsymbol{\Sigma}$ **△ ↑「貝ノ」 C ↑HV L A △ 不值欠了 乙不口 \$,** \mathbb{R}^n ! = **.** ! O
→ U ^anαⁿ|n# L d²α ! !

n

!

n=N

'

!

 $T_{\rm eff}$ is integral over the area $T_{\rm eff}$ is well-defined (as well-defined (as $T_{\rm eff}$

$$
\begin{array}{ll}\n\text{Theorem 18, arXiv:2402.0831} \\
\text{(i)} & \sum_{n=0}^{\infty} |a_n|^2 |\alpha|^{2n} < \infty, \quad \alpha \in D(r), \\
\text{(ii)} & \int_{D(r)} \sum_{n=0}^{\infty} |a_n|^2 |\alpha|^{2n} d^2 \alpha < \infty. \\
\text{(iii)} & \int_{D(r)} \sum_{n=0}^{\infty} |a_n|^2 |\alpha|^{2n} d^2 \alpha < \infty. \\
\text{(iv)} & \int_{D(r)} \sum_{n=0}^{\infty} |a_n|^2 |\alpha|^{2n} d^2 \alpha < \infty. \\
\text{(v)} & \int_{D(r)} \sum_{n=0}^{\infty} |a_n|^2 |\alpha|^{2n} d^2 \alpha < \infty. \\
\text{(v)} & \int_{D(r)} \sum_{n=0}^{\infty} |a_n|^2 |\alpha|^{2n} d^2 \alpha < \infty. \\
\text{(v)} & \int_{D_{k=0}}^{\infty} |a_k|^2 |\alpha|^{2k} d^2 \alpha < \infty. \\
\text{(v)} & \int_{D_{k=0}}^{\infty} |a_k|^2 |\alpha|^{2k} d^2 \alpha < \infty. \\
\text{(v)} & \int_{D_{k=0}}^{\infty} \sum_{n=0}^{\infty} \int_{D} |a_k|^2 |\alpha|^{2k} d^2 \alpha < \infty. \\
\text{(v)} & \int_{D_{k=0}}^{\infty} \sum_{n=0}^{\infty} a_n \alpha^n |n \rangle d^2 \alpha < \mathcal{H} \\
\text{(v)} & \int_{D_{k=0}}^N |\alpha - \psi| < ||\phi - \psi|| \le ||\phi - \phi^{(N)}|| + ||\phi^{(N)} - \psi^{(N)}|| + ||\psi^{(N)} - \psi|| \\
\text{(v)} & \int_{D_{k=0}}^N \sum_{n=0}^N a_n \alpha^n |n \rangle d^2 \alpha < \mathcal{H} \\
\text{(v)} & \int_{D_{k=0}}^N |\alpha - \psi| < ||\phi - \psi|| \le ||\phi - \phi^{(N)}|| + ||\psi^{(N)} - \psi|| \\
\text{(v)} & \int_{D_{k=0}}^
$$

'

<u>(ii) #1</u>

Therefore, the state vector in the form of

積分と和の交換 $\tau \pm \tau$ n見刀 ⊂ 1´H V ズ] ズ mal basis on H. Let be D(r) := { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C =
{ α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C = { α ∈ C =
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→ U ^anαⁿ|n# L d²α ! !

n

!

n=N

'

!

 $T_{\rm eff}$ is integral over the area $T_{\rm eff}$ is well-defined (as well-defined (as $T_{\rm eff}$

$$
\begin{array}{ll}\n\text{Theorem 18, arXiv:2402.0831} \\
\text{(i)} & \sum_{n=0}^{\infty} |a_n|^2 |\alpha|^{2n} < \infty, \quad \alpha \in D(r), \\
\text{(ii)} & \int_{D(r)} \sum_{n=0}^{\infty} |a_n|^2 |\alpha|^{2n} d^2 \alpha < \infty. \\
\text{(iii)} & \int_{D(r)} \sum_{n=0}^{\infty} |a_n|^2 |\alpha|^{2n} d^2 \alpha < \infty. \\
\text{(iv)} & \int_{D(r)} \sum_{n=0}^{\infty} |a_n|^2 |\alpha|^{2n} d^2 \alpha < \infty. \\
\text{(v)} & \int_{D(r)} \sum_{n=0}^{\infty} |a_n|^2 |\alpha|^{2n} d^2 \alpha < \infty. \\
\text{(v)} & \int_{D(r)} \sum_{n=0}^{\infty} |a_n|^2 |\alpha|^{2n} d^2 \alpha < \infty. \\
\text{(v)} & \int_{D} \sum_{n=0}^{\infty} |a_n|^2 |\alpha|^{2n} d^2 \alpha < \infty. \\
\text{(v)} & \int_{D} \sum_{n=0}^{\infty} |a_n|^2 |\alpha|^{2n} d^2 \alpha < \infty. \\
\text{(v)} & \int_{D} \sum_{n=0}^{\infty} |a_n|^2 |\alpha|^{2n} d^2 \alpha < \infty. \\
\text{(v)} & \int_{D} \sum_{n=0}^{\infty} |a_n|^2 |\alpha|^{2n} d^2 \alpha < \infty. \\
\text{(v)} & \int_{D} |a_n|^2 \leq |D| \sum_{n} \int_{D} |a_n \alpha^n|^2 d^2 \alpha < \infty. \\
\text{(v)} & \int_{D} \sum_{n=0}^{\infty} |a_n|^2 |\alpha|^{2n} d^2 \alpha < \infty. \\
\text{(v)} & \int_{D} \sum_{n=0}^{\infty} |a_n|^2 |\alpha|^{2n} d^2 \alpha < \infty. \\
\text{(v)} & \int_{D} \sum_{n=0}^{\infty} |a_n|^2 |\alpha|^{2n} d^2 \alpha < \infty. \\
\text{(v)} & \int_{D} \sum_{n=0
$$

'

<u>(ii) #1</u>

Next, let us define

Therefore, the state vector in the form of

- Riemann 積分:一様収束を証明すればよい。
- where in the last line we use $\frac{1}{2}$ in $\frac{1}{2}$ in Appendix Eq. () in Appendix Eq. () in $\frac{1}{2}$ • Lebesgue 積分:優収束定理などを使用できる。
- $r=\pm 1$. Depeab^t $\frac{1}{2}$ is unit $\frac{1}{2}$ is uniformly bounded. • Bochner 積分 : Banach空間に値をとる積分 "
|
|
| יי し個 e−|α[|] 優収束定理を使用できる √ ベクトル値 $L_2[\mathbb{C}, \ell_2]$ α 。

 :
:1 \overline{r} e_2 [C, $\mathscr{B}(\mathscr{D})$ 作用素値 *L*₂[C, $\mathscr{B}(\mathscr{H})$] $\int \frac{\ln |1 - \frac{1}{2}|}{\ln |1 - \frac{1}{2}|}$ m! $\frac{1}{2}$ $L_2[\mathbb{C}, \ell_2]$ $\qquad \qquad |||a\rangle\langle a|\varphi\rangle||^2$ *d*2 *α* < ∞ ℋ値可積分 [∫] [∥]|*α*⟩⟨*α*|∥² *d*2 *α* = ∞ 作用素値可積分でない • ノルムが可積分なら存在確定

数列空間で作業していたのに、関数空間の知識が必要になるのは残念

スモールエル2で作業していたのに、ラージエル2

Klauder の講義ノートによる別証明

Suggested Problems 1.5 (2006年)

http://www.phys.ufl.edu/~klauder/norway/

$$
\mathbf{E} \mathbf{E} \qquad \mathbf{0} \leq A_n \leq I \; \mathbf{b} \; \mathbf{D} \; A_n \leq A_{n+1} \; \mathbf{b} \; \mathbf{D} \qquad \lim_{n \to \infty} \langle \phi | I - A_n | \psi \rangle = 0
$$

$$
\Rightarrow \lim_{n \to \infty} ||(I - A_n)\psi|| = 0
$$

$$
\langle \phi | A_n | \varphi \rangle := \pi^{-1} \int_{|\alpha| \le n} \langle \phi | \alpha \rangle \langle \alpha | \varphi \rangle d^2 \alpha,
$$

として、弱収束を証明する。 上記の定理の条件を満たすことを示せば、強収束が証明できる

- 単調性があれば弱収束から強収束へ持っていける
- 射影でない族でもよい

まとめ R. Namiki arXiv:2402.0831

- 作用素の収束、3種類
- 完全性条件は強収束
- 初等的な証明
- Bochner積分は便利
- Klauderによる別証明

$$
|\phi\rangle \frac{S}{=} \frac{1}{\pi} \int_{\alpha \in \mathbb{C}} |\alpha\rangle \langle \alpha | \phi \rangle d^2 \alpha \qquad \text{c.f.} \quad |\phi\rangle = \sum_{n=0}^{\infty} |n\rangle \langle n | \phi \rangle
$$

$$
|\alpha\rangle = e^{-|\alpha|^2/2} \sum_{n=0}^{\infty} \frac{\alpha^n}{\sqrt{n!}} |n\rangle, \quad \alpha \in \mathbb{C} \qquad I \stackrel{\text{s}}{=} \sum_{n=0}^{\infty} |n\rangle \langle n|
$$