Construction and characterization of 1-parameter (non)decomposable maps

Abstract

Y. Yang et al. [1] showed that every *2*-positive map working with M_3 on itself is decomposable. This is an extension of the known fact that every positive map acting with M_2 on itself is decomposable [2,3]. Moreover, it is known that all the positive maps from M_2 to M_3 also have the decomposability property [3]. Therefore, the following questions seem natural:

1. Is every (n - 1)-positive map acting on M_n into itself decomposable?
2. Is every (n - 1)-positive map acting on M_n into M_{n+1} decomposable?

The above questions can also be expressed as questions about states on systems composed of two subsystems. Namely, these are questions about the maximum Schmidt number for PPT states.

A weaker version of the above questions is the question about the existence of indecomposable *k*-positive maps, where $k \geq 2$. There are known examples of *2*-positive indecomposable maps in the higher dimensions [4], but we still don't know if such maps exist in the lower dimensions. Giving examples of indecomposable *k*-positive maps and developing a method for constructing such maps would explain the relationship between entanglement and the Schmidt number for states in complex systems [4,5].

The result of research is a specific construction of positive maps acting between matrix algebras of any dimensions [6]. We found that, with k being fixed for some controllable parameter, the maps we constructed are *k*-positive and not $(k+1)$-positive, and not completely copositive. This would suggest that these maps are not decomposable. Moreover we found characterization of positivity i.e. positivity, *k*-positivity and completely positivity of our maps in special case where the dimension of the domain and image differ by one. Additionally we discover behaviour of our map that for special dimensions that holds $n \geq 3m - 2$ then completely copositivity is guaranteed whenever controllable parameter is more or equal to dimension of the domain.

Literature:

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